EE 367000 Introduction to Mathematics for Communications: Convex Analysis and Optimization

(通訊之數學導論:凸分析與優化) Spring 2019

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Units: 3

Lecture hours: T3, T4, Th3, Th4

Classroom: **Delta 210**Course web page:

 $\underline{https://www.ee.nthu.edu.tw/cychi/teaching/Introduction-to-Mathematics-for-Communications-Convex.php}$

Office hours: Thursday 13:30-15:30

Course Description, goals and expectations:

This course aims to introduce *convex analysis and optimization* for undergraduate students for pursuing their graduate studies by doing cutting-edge research. Over the last two decades, convex optimization has been recognized as a powerful mathematical tool for solving wide range of optimization problems in science and engineering, such as (a) blind source separation (BSS) for biomedical and hyperspectral image analysis, (b) computer vision and machine learning, and (c) multiple-input multiple-output (MIMO) wireless communications and networking (i.e., coherent/noncoherent detection, transmit/robust/distributed beamforming, and physical-layer secret communications). The foundation of convex optimization is convex analysis which is a branch of mathematics that studies properties of convex sets and convex functions.

The first part of this course will be dedicated to review fundamental concepts of linear algebra and then expose students to more advanced topics in linear algebra and matrix theory. The concepts, tools and specifically the language of linear algebra are absolutely essential and widely used in engineering, physics, economics, social sciences, and natural sciences. Here, we will use linear algebra to translate a described optimization problem into mathematical formulations for further analysis. Moreover, selected topics from advanced calculus and real analysis will be introduced. The second part of the course will be focused on: *how to reformulate an optimization problem into a convex problem and then solve it optimally*.

The main goals of this course can be summarized as follows:

- To review the backgrounds of basic Linear Algebra and Calculus
- To teach the important concepts in Advanced Linear Algebra, Matrix Theory and Advanced Calculus
- To expose students to the fundamentals of Convex Analysis and Optimization
- To address some applications of this powerful mathematical tool

This course not only discusses fundamentals of convex analysis and optimization to strengthen the students' mathematical background but also provides an opportunity for students to train their mind to think and discuss with logical reasoning. This can lead them to learn: "how to do and write a neat mathematical proof?" To this end, students are expected to actively exercise writing mathematical proofs to make their own proofs in the homework assignments.

Expectations: Empowering students through convex analysis and convex optimization concepts and level up their skills to conduct their future research projects with foundations which are suitable in a wide range of applications in not only communication engineering but also signal processing, electrical engineering, computer

science and economics.

Background & Prerequisite: A good background in linear algebra and calculus is desirable. **Outline:**

- 1. Review on Basics of Linear Algebra, Set Theory and Geometry/Topology
- 2. Advanced Topics in Linear Algebra, Matrix Theory and Calculus
- 3. Convex Sets
- 4. Convex Functions
- 5. Convex Optimization Problems
- 6. Geometric Programming
- 7. Duality (introductory level)

Textbooks:

- Chong-Yung Chi, Wei-Chiang Li, and Chia-Hsiang Lin, Convex Optimization for Signal Processing and Communications: From Fundamentals to Applications, CRC Press, Boca Raton, FL, 2017. http://st-ebook.com.tw/bookcomment-2.aspx?BOKNO=TKCP00033 (科文化圖書公司)
- † The material of the textbook systematically introduces how to efficiently and effectively solve an optimization problem, from the fundamental theory, problem definition, reformulation into a convex problem, analysis, algorithm implementation, to cutting edge research in signal processing and communications (like an exploration journey rather than pure mathematics). It has been used for my 2-week (32 lecture hours) or 3-week (48 lecture hours) invited short course entitled "Convex Optimization for Signal Processing and Communications" at many top ranked universities in Mainland China over the last decade, including Shandong University, Jinan (January 2010, November 2017), Tsinghua University, Beijing (August 2010 and August 2012), Tianjin University, Tianjin (August 2011), Beijing Jiaotong University (BJTU), Beijing (July 2013, July 2015 and August 2017), University of Electronic Science and Technology of China, Chengdu (November 2013, September 2014 and September 2015), Xiamen University, Xiamen (December 2013), Sun-Yet-Sen University (SYSU), Guangzhou (August 2015), and Beijing University of Posts and Telecommunications, Beijing (July 2016, July 2017, and July 2018), and Shandong Normal University, Jinan (Aug. 2018).
- S. H. Friedberg, A. J. Insel, and L. E. Spence, Linear Algebra, 4th ed., Prentice Hall, 2003.

References:

- S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge: Cambridge University Press, 2004. Free electronic version is available at: http://www.stanford.edu/~boyd/cvxbook/.
- R. A. Horn and C. R. Johnson, Matrix Analysis, 2nd ed. Cambridge: Cambridge University Press,
- C. H. Edwards, Advanced Calculus of Several Variables, Academic Press, 1973.
- Giuseppe Calafiore and Laurent El Ghaoui, Optimization Models, University Press, Cambridge, 2014.
- D. P. Bertsekas, Convex Analysis and Optimization, Athena Scientific, 2003.

Grading:

Your total score out of 110 points is allocated as:

- Homework: **60 points plus 10 points** for creative solutions;
- Final exam: 40 points; written examination in class.

TAs:

• PingRui Chiang, e-mail: <u>a0988731275@gmail.com</u>; **Office hours:** Wednesday 14:00-16:00. (Office: EECS Building 706, Tel: X34033)

• Sadid Sahami, e-mail: sadid@gapp.nthu.edu.tw; Office hours: Thursday 16:00-18:00. (Office: EECS Building 706, Tel: X34033)

Notice:

- 1. Your course grade will be based on a nonlinear adjustment on the total score which will be determined after the final examination.
- 2. No make-up for final examinations under any circumstance.